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# Microbubble Drag Reduction Model considering Pressure Gradient

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The new theoretical model [1] to evaluate the microbubble drag reduction due to the presence of bubbles inside the boundary layer where there exists a pressure gradient as external boundary condition of the boundary layer will be presented. For the development of this theoretical model, we assume that the bubbles remains in a sublayer of the boundary layer as shown in Fig. 1 and we begin by describing the properties of the boundary layer using the traditional Von Karman integral equation, assuming that the velocity outside the boundary layer changes along the tangential component to the surface. Then, we assume the following hypotheses in order to be able to model the drag reduction:

1. In analogy with the flat plate boundary layer problem we assume that the velocity inside the boundary layer scaled with the velocity outside the boundary layer at each tangential position is a self-similar quantity by a power-law dependency on the normal position scaled with the boundary layer thickness.
2. The density can be expressed from a self-similar quantity when it is scaled through the density deficit.

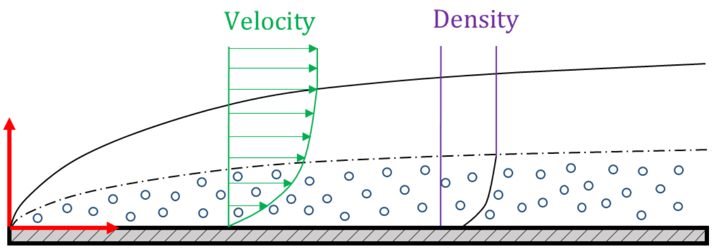


Figure 1: Boundary Layer.

As a result, drag reduction can be expressed as the sum of three terms related to the pressure gradient, the wall void fraction, and the local gradient of the wall void fraction. The effect that the boundary layer structure has on each of the drag reduction terms will be discussed and maps that show the regions where there is a drag reduction in the parametrical space considered will be presented.
Based on the theoretical analysis conducted for each of these terms, it has been concluded that in order to make the pressure gradient beneficial for drag reduction, provided there is sufficient void fraction within the boundary layer, the bubbles should be present in regions where there is a positive pressure gradient, and the void fraction should increase along the chord. Conversely, if there is insufficient void fraction, the bubbles will be beneficial for drag reduction in regions with a negative pressure gradient.
These results could help to assess the applicability of the microbubble drag reduction technique to marine propellers where there appears a pressure gradient as contraposition to what happens in the hulls. However, all the conclusions drawn in this study are based on theoretical assumptions, and experimental results are required to confirm these findings.

**Keywords:** Microbubble Drag Reduction, Boundary Layer, Multiphase, Bubbles Dynamics.

**References**:

[1] Garcia-Magariño et al., *Ocean Engineering,* **2025**. https://doi.org/10.1016/j.oceaneng.2025.120797